Government General Degree College, Chapra

Department of Mathematics

Project Name- Find the solution of a system of linear equations using reduced row echelon form of a matrix

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Row Echelon Form of a Matrix:-

A matrix is in row echelon form if the following conditions are satisfied:-

1. Rows consisting entirely of 0's occur at the bottom of the matrix.

2. The first entry in any row with nonzero entries is 1.

3. The column subscript of the leading 1 entries increases as the row subscript increases.

Example:-

$$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Reduced Row Echelon Form:-

We say that a matrix is in reduced row echelon form if and only if it is in row echelon form, all its pivots are equal to 1 and the pivots are the only non-zero entries of the basic columns.

Example:-

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

1. Use elementary row operations to reduce the matrix to row-echelon form, and hence determine the rank of the matrix

$$A = \begin{bmatrix} 4 & 7 & 4 & 7 \\ 3 & 5 & 3 & 5 \\ 2 & -2 & 2 & -2 \\ 5 & -2 & 5 & -2 \end{bmatrix}$$

2. Reduce the matrix to reduced row-echelon form and hence determine the rank of the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 3 & -2 & 0 & 7 \\ 2 & -1 & 2 & 4 \\ 4 & -2 & 3 & 8 \end{bmatrix}$$

3. Use Gauss - Jordan elimination to determine the solution set to the system:-

$$x_1 + x_2 + x_3 - x_4 = 4$$
$$x_1 - x_2 - x_3 - x_4 = 2$$
$$x_1 + x_2 - x_3 + x_4 = -2$$
$$x_1 - x_2 + x_3 + x_4 = -8.$$

Apply elementary row operations to reduce the following matrix to a row echelon matrix.

$$\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$$

$$\begin{array}{l} R_2-3R_1 \\ R_3-5R_1 \end{array} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 3 & 2 & 5 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 3 & 2 & 5 \end{pmatrix}.$$

$$\begin{array}{l} R_3-2R_2 \\ R_4-3R_2 \end{array} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} \xrightarrow{R_{34}} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{2}R_3} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1-2R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = R, \text{ say.}$$

R is a row echelon matrix.

Working Procedure. The element in the (1,1) position is 2.

Step 1. Multiply the 1st row by $\frac{1}{2}$. The leading element in the first row becomes 1. in the (1,1) position.

Step 2. To reduce all other elements in the first column to zero, perform the operations R_2-3R_1, R_3-5R_1.

Consider the submatrix obtained by deleting the first row and the first column. Observe that the element in the (1,1) position of the submatrix. i.e, the element in the (2,2) position of the original matrix is 2.

Step:-3. Multiply the second row by $\frac{1}{2}$. The leading element in the second row becomes 1 in the (2,2) position.

As we saw in The Matrix and solving systems using Matrices section, the reduced row echelon form method can be used to solved systems.

With this method, we put the coefficients and constants in one matrix (called and augmented matrix, or in coefficient form) and then, with a series of row operations change it into what we call reduced echelon form, or reduced row echelon form. This is when the leading entry in every nonzero row is 1, and each leading 1 is the only nonzero entry for that column, such as.

$$\begin{bmatrix} 1 & 0 & 0 & \text{"x"} \\ 0 & 1 & 0 & \text{"y"} \\ 0 & 0 & 1 & \text{"z"} \end{bmatrix}$$ Then the numbers in the last column are the answers!

This is also called Row Reduction.